Analytisch-Physikalische Chemie

Teil Physikalische Chemie
Sommersemester 2007

Verifying Ohm's Law and characterisation of a RC low-pass filter (ELK)

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Abstract: In the following experiment familiarising with basic electronic circuits was the ultimate aim. Using an oscilloscope and a frequency generator, three different types of signal were observed, a rectangular, a triangular and a sinus like frequency. The most accurate measurement in both the time and the volt could be observed when using a frequency in the kHz range, and by choosing an appropriate divider, so the complete loop could be observed on the oscilloscope.
Ohm's Law could be verified in two different experiments, by connecting two resistors in series and by varying the voltage and in the second the resistor.
The border frequency of a RC-circuit consisting of a 10 kΩ resistance and a 10 nF capacitor was determined to be 1412 Hz. The theoretical value for the border frequency is 1684 Hz.
The only figure which was observed was the figure at a frequency-ratio of 1:2.

Zürich ZH, 15. Mai 2007

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1. Introduction

1.1. Theory

Periodic voltage and alternating current

The voltage U is the difference of electrical voltage between two points in an electrical circuit and is measured in Volt. The current I describes the speed of the flow of electrical charges. The unit of electrical current is Ampere. The electrical current exists as direct current and alternating current.

Periodic voltage and alternating current can be described by

$$U(\omega, t) = U_0 \cos(\omega t + \phi_u) = U_0 \exp(i\omega t) \quad (1)$$
$$I(\omega, t) = I_0 \cos(\omega t + \phi_I) = I_0 \exp(i\omega t) \quad (2)$$
$$\text{with} \quad \omega = 2\pi f \quad (3)$$

U_o and I_o are the amplitudes, ϕ_u and ϕ_I the phases.

Ohm´s law

The Ohm´s resistor is defined by following equation which gives the relation between the current and the voltage

$$R = \frac{U}{I} \quad (4)$$

R is the resistor with the unit Ω.

Is case of serial or parallel connection the total equivalent resistance is

$$\text{Serial:} \quad R = \sum_i R_i \quad (5)$$
$$\text{Parallel:} \quad \frac{1}{R} = \sum_i \frac{1}{R_i} \quad (6)$$

A voltage divider is commonly used to create a certain voltage from another proportional voltage. The input voltage is related to the output voltage as shown in the equation.

$$U_2 = \frac{R_2}{R_1 + R_2} U \quad (7)$$

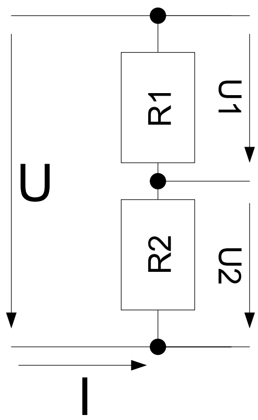


Figure 1: A voltage divider created by connecting two resistors.

Capacity

A capacitor is an electrical device that stores energy in an electric field between two plates (conductors). When voltage is applied charges with opposite charge built up at the conductors. The relation between the charge Q and the voltage V is given by

$$Q = CU \quad \textbf{(8)}$$

Whereas C is the capacity with the unit As V^{-1}=F (Farrad).

The Voltage at the capacitor follows the incoming current with a phase shift of 90°.

Low-pass filter

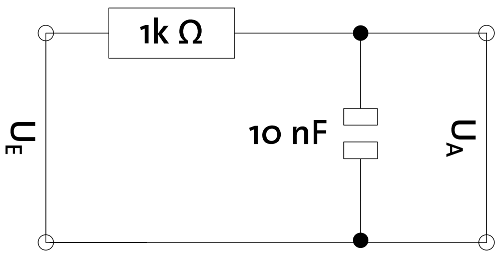


Figure 2: Circuit diagram of a low-pass filter

The low-pass filter consists of a resistor in serial configuration and a capacitor in serial configuration. It attenuates high frequencies but let low frequencies pass. The complex voltage gain function for this case is:

$$G(\omega) = \frac{1}{1 + i\omega RC} \quad \textbf{(11)}$$

The border-frequency f_g is a characteristic quantity which describes the frequency where the initial intensity is reduced to its half. The frequency of the corresponding 3-dB-point (border frequency) can be calculated with following equations.

$$\omega_g = \frac{1}{RC} \text{ (12) resp. } f_g = \frac{1}{2\pi RC} \quad \textbf{(13)}$$

The phase difference ϕ_g is

$$\tan(\phi_g) = \frac{Im\{G(\omega)\}}{Re\{G(\omega)\}} = -\omega RC \quad \textbf{(14)}$$

Bode-plot

A Bode-plot is a graph of the amplitude and the phase shift, both as a function of the frequency. The frequency range is of log magnitude and range of voltage is also of log magnitude in dB using the relation:

$$A = 10 \log \frac{|U_{out}|}{|U_{in}|}, \text{ [A]=dB} \quad \textbf{(15)}$$

Lissajous figures

If two signals are plotted with the help of an oscilloscope, the resulting figure is called Lissajous figures. If both signals are at the same phase, meaning that both have the same frequency a standing picture of a circle could be observed on the oscilloscope. If the ratio between the two frequency is rational, like 1:2 or 1:3, the resulting Lissajous-figure is a certain standing image.

2. Experiment

2.1. Experimental setup

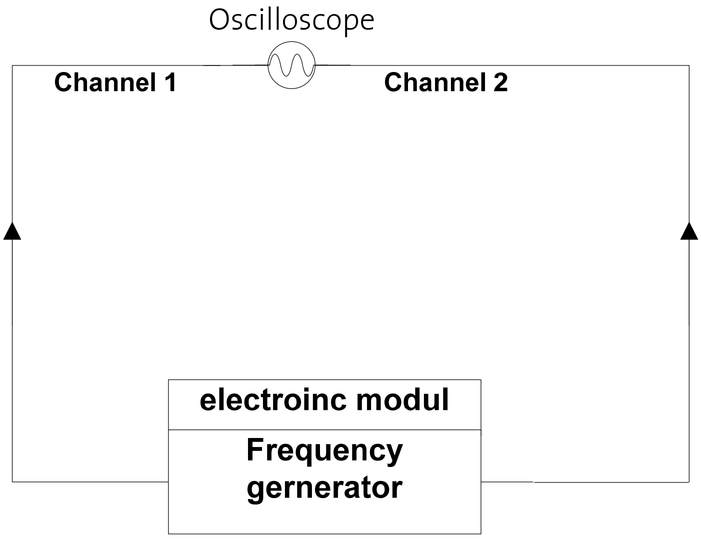


Figure 3: Schematically picture of the experimental setup for electric experiment.

The used setup is presented in Abb. 2. The signal from the frequency generator is spitted in to two signals, one signal is connected directly to the oscilloscope (KIKUSUI DSS 5040), the other was sent over the electronic modules from the Lehrwerkstätte Bern and then to the second channel to the oscilloscope. The oscilloscope is able to show both channels at the same time, as well as show them separately. The frequency counter and the frequency generator (MS-9140) is able to count the frequency in the range of 0 to 250 MHz, the generator part can generates three different types of curves, a sinus ideal, a triangular and a rectangular one., the frequency can be varied from 0.02 Hz to 2 MHz, the output amplitude can be varied from 0 V to 10 V. To measure the resistance a digital multimeter (MS-9140) was used.

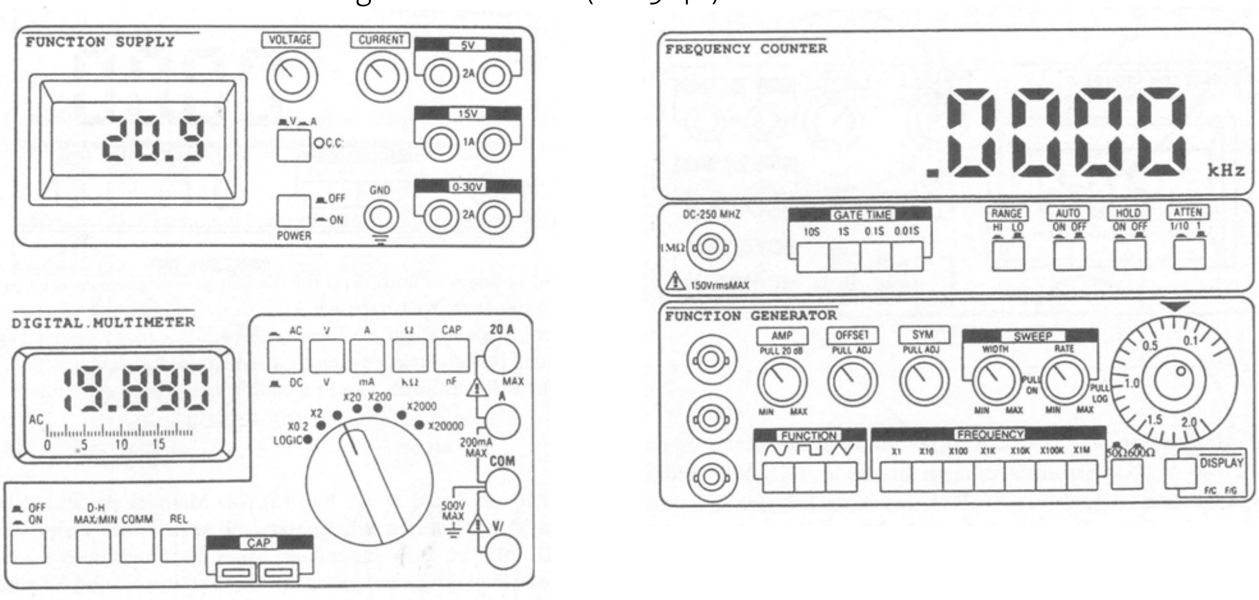


Figure 4: Schematically picture of function supply, frequency generator and the Digital Multimeter [1].

2.2. Execution of the experiment

2.2.1. Familiarization

The first part of the experiment was to familiarize with the oscilloscope and the function generator. The goal was to detect what different signal the function generator generates and how they are displayed on the oscilloscope.
The second part was to increase the accuracy, there for different sinus-like frequencies were generated and than analyzed by measuring the time domain, and calculating the error.

2.2.2 Ohm's Law

The second part of the experiment was to verify Ohm's law. In the first subpart, two resistors (R1 = 1kΩ and R2 = 10kΩ) were connected in series, the potential difference between R2 were measured, while the entrance Voltage was varied. In the second sub part, the voltage was kept at a constant value, while the 10kΩ – Resistor is variable.

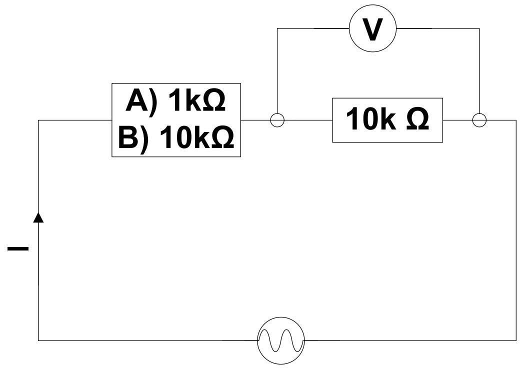


Figure 5: Circuit, which was used to verify Ohm's law.

2.2.3 Characterization of a RC low-pass filter

A capacitor (C = 10 nF) and a Resistor (R = 10kΩ) were placed in series. The frequency is varied in a logarithmic scale from 0.5 kHz up to 0.5 MHz. The out coming voltage and the phase difference between the two signals were noted down.

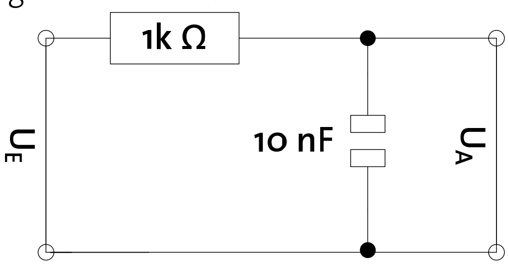


Figure 6: Schematically circuit, which was used for the characterization of a RC low-pass filter

2.2.4 Lissajous-figures

The signals from two different frequency generators were plotted in x and y direction on the oscilloscope, by choosing certain proportion of the frequency (eg. 1:1, 1:2...) the different lissajous figures could be observed.

3. Analysis and Results

3.1. Familiarization

Building up the described circuit didn't provide any problem. By pressing the adequate button on the function-generator the three different forms of function, which are shown in figure 7, could be observed.

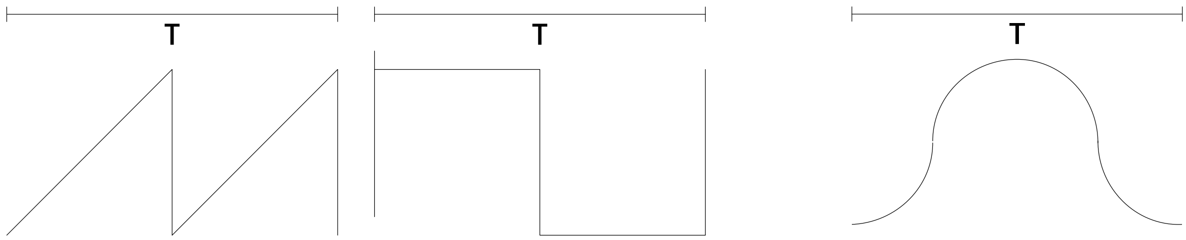


Figure 7: The observed typed of function, a sinus like, a rectangular and a triangular like.

The accuracy of the time dimension could be increased by showing a half wave on the whole screen, by choosing an appropriate division factor and using frequency in the upper kHz region.

f in [Hz]	Periode [s]	Calculated f	Error
1025.656	0.0009800	1020.40816	8.14%
10181.8	0.0000980	10204.0816	4.09%
102078.9	0.0000096	104166.667	4.25%
1163202	0.0000009	1111111.11	4.25%

Tab. 1: Measured period and frequency, and the calculated error.

The accuracy volt-scale could be increase by using the smallest division factor, so the top of the wave could still be observed at the oscilloscope.

3.2. Ohm's Law

The data which were used for figure 8 and 9 can be found in the appendix.

3.2.1 Variable Voltage

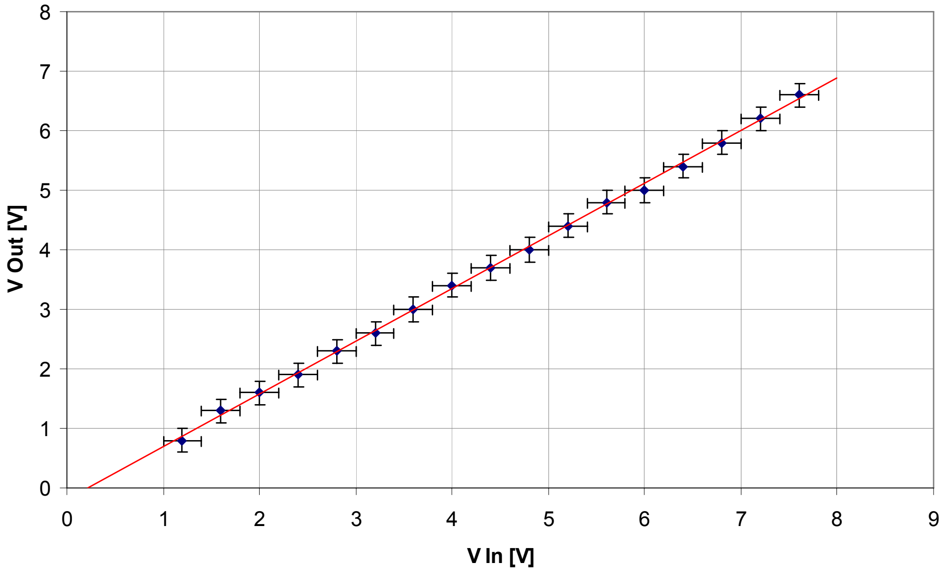


Fig. 8. Plot of the Voltage Task 2.2.2 part A

The linear regression results y = 0.8836x - 0.1936, with R^2=0.9991. The slope is 0.8836, this does correspond to result of $\frac{R_1}{R_1 + R_2}$, with R_1=10Ω, and R_2=1Ω.

3.2.1 Variable Resistance

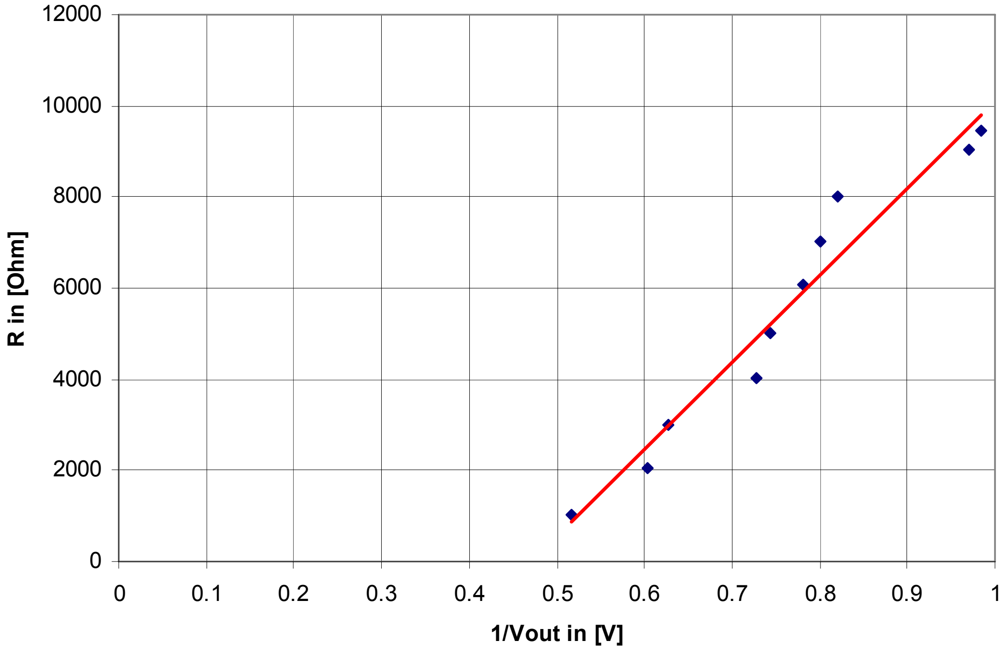


Fig. 9. Plot of the Voltage Task 2.2.2 part B

The linear regression results y = 19073x - 8972.7 and R^2 = 0.9514. Resistor number 2 is 9.0 kΩ and $R_2 \cdot V_{out}$ is 19.1 kΩ·V.

3.3. RC low-pass filter

The data, which were used in figure 10 and 11 can be found in the appendix.

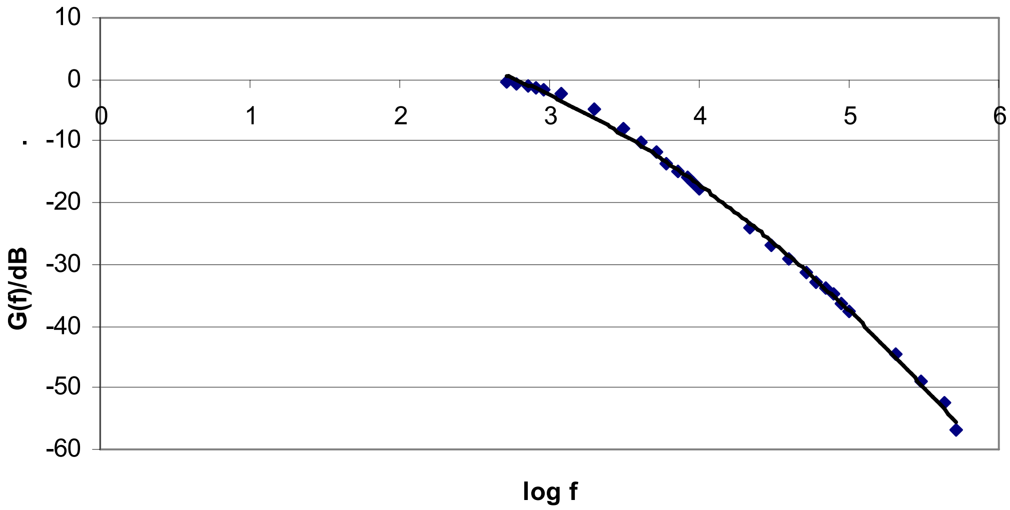


Figure 10: Standardized Bode-plot for the low-pass filter. The graphically obtained barrier frequency f_g is 1412 Hz.

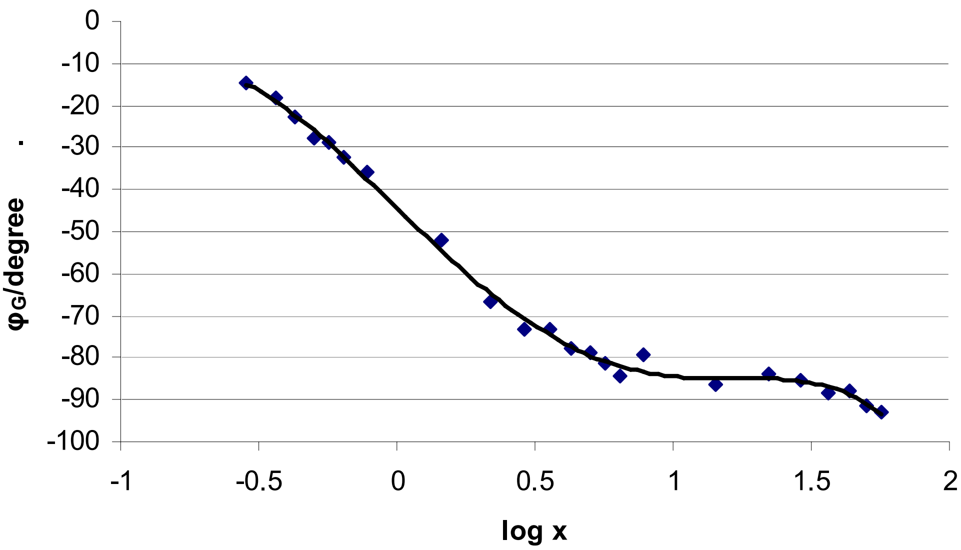


Figure 11: Plot of the phase shift versus the standardized frequency in logarithmic scale.

3.4 Lissajous-figures

The frequency generator generated a frequency about 315 Hz, by displaying the signal from channel 1 versus the signal from the channel 2 (x-y-modus) the corresponding frequency on the second generator was search. When a circle was observed, the frequency on both generators was the same as displayed on the frequency counter. By doubling the frequency on the first generator the following figure could be detected.

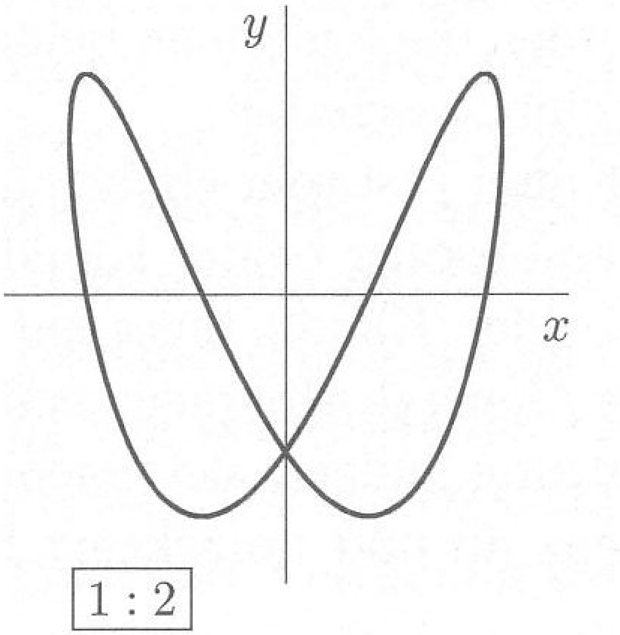


Figure 12: The observed lissajous-figure at the frequency proportion of 1 :2 [1]

4. Discussion

4.1 Familiarization

The setting up the frequency generator and the oscilloscope didn't provide any problems. All three different waveforms could be easy observed and charactericied. As shown the accuracy could be improved by using a frequency in the upper kHz region (e.g. 100 kHz), the most important improving is by always choosing the best division, so the read value has the biggest accuracy.

4.2 Ohm's Law

4.2.1 Variable Voltage

The linear regression returned: y = 0.8836x - 0.1936, The Value 0.8836 corresponds to the result of $\frac{R_1}{R_1 + R_2}$, of which the calculated value (from the theoretical values) is 0.91, so the error is about 3%.

4.2.2 Variable Resistor

Out of the linear regression R2 was determined to be 9.0 $k\Omega$, this about the value which was displayed on the Resistor (10 $k\Omega$). The error could be either because the resistor has a aberration of about 20%, the second was due to read-off error, when determining the voltage and determining the resistance of R_1

4.3 RC Low-pass Filter

The determined value of 1412 Hz for f_g from graph x does not match exactly with theoretical value of 1684.18 Hz. A reason for this difference is that the graphical determined value is only estimation. It was not possible to find a more precise value because the measurement started at 500 Hz. Without lower frequencies one could not deduce f_g very well.
Figure 11 presents the function of a low-pass filter. The graph shows that low frequencies can pass and that high frequencies are attenuated.
The plot of the phase shift versus the frequency gives a value for ω_g around 45° which matches with the theory. High frequencies have a phase shift of 90°, low frequencies of 0°.

4.4 Lissajous-figures

Because of some problems with the coaxial cable the lissajous-figures could not be observed very clear, there were instead of one circle, 4 parallel circles. Other problems did not occur. Only one figure was detected, so the other group could use the frequency generator as well.

5. References

[1] Meister, E., *Grundpraktikum in physikalischer Chemie – Theorie und Experimente*, VDF Hochschulverlag, Zürich, 2006.

6. Appendix

Raw Data for Task 1 – Accuracy check

f in Hz	**Periode /s**	**Calculated f**	Error read off	Min f	Max f	Differenc	**Error**
1025.656	0.0009800	1020.40816	0.00004	980.392157	1063.82979	83.4376304	8.14%
10181.8	0.0000980	10204.0816	0.000002	10000	10416.6667	416.666667	4.09%
102078.9	0.0000096	104166.667	0.0000002	102040.816	106382.979	4342.1624	4.25%
1163202	0.0000009	1111111.11	0.00000002	1086956.52	1136363.64	49407.1146	4.25%

Raw Data for Task 2 Part A

Vout [V]	Vin [V]	Error in Vin	Error in Vout	Axis intercept	Slope
0.8	1.2	0.2	0.2	**-0.193627451**	**0.883578431**
1.3	1.6	0.2	0.2		
1.6	2	0.2	0.2		
1.9	2.4	0.2	0.2		
2.3	2.8	0.2	0.2		
2.6	3.2	0.2	0.2		
3	3.6	0.2	0.2		
3.4	4	0.2	0.2		
3.7	4.4	0.2	0.2		
4	4.8	0.2	0.2		
4.4	5.2	0.2	0.2		
4.8	5.6	0.2	0.2		
5	6	0.2	0.2		
5.4	6.4	0.2	0.2		
5.8	6.8	0.2	0.2		
6.2	7.2	0.2	0.2		
6.6	7.6	0.2	0.2		

Raw Data for Task 2 Part B

Resistor 1 / Ω	**Voltage error / V**	**Vout [V]**	**Vin [V]**	1/Vout [V]
1044	±0.04	1.94	2	0.51612903
2037	±0.04	1.66	2	0.60377358
2997	±0.04	1.59	2	0.62745098
4040	±0.04	1.38	2	0.72727273
5020	±0.04	1.34	2	0.74418605
6070	±0.04	1.28	2	0.7804878
7040	±0.04	1.25	2	0.8
8000	±0.04	1.22	2	0.82051282
9040	±0.04	1.03	2	0.96969697
9450	±0.04	1.02	2	0.98461538

Raw Data from Task 3

Calculated values for the phase difference and standardized frequency with f_g=1412 Hz.

f/Hz	Δt/s	φ/degree	Log(x)
80667,70	0,0000032	-92,92919040	1,756864978
70654,21	0,0000036	-91,56785616	1,699303348
60954,88	0,0000040	-87,77502720	1,635173784
51100,76	0,0000048	-88,30211328	1,558592663
40920,70	0,0000058	-85,44242160	1,462108358
31057,90	0,0000075	-83,85633000	1,342337391
20016,40	0,0000120	-86,47084800	1,151551274
11000,70	0,0000200	-79,20504000	0,891585624
9023,45	0,0000260	-84,45949200	0,805537919
8064,46	0,0000280	-81,28970640	0,756740326
7065,24	0,0000310	-78,84807840	0,699292222
6018,00	0,0000360	-77,99328000	0,629617487
5091,11	0,0000400	-73,31198400	0,556977784
4072,00	0,0000500	-73,29600000	0,459973073
3088,00	0,0000600	-66,70080000	0,339842595
2054,80	0,0000700	-51,78096000	0,16293486
1099,87	0,0000900	-35,63578800	-0,10849334
904,03	0,0001000	-32,54508000	-0,19365185
805,96	0,0001000	-29,01456000	-0,24352121
702,99	0,0001100	-27,83842776	-0,30288518
601,50	0,0001050	-22,73665464	-0,37059993
510,28	0,0001000	-18,37008000	-0,44202615
404,53	0,0001000	-14,56296480	-0,5428874

Values for the standardized Bode-plot of the low-pass filter.

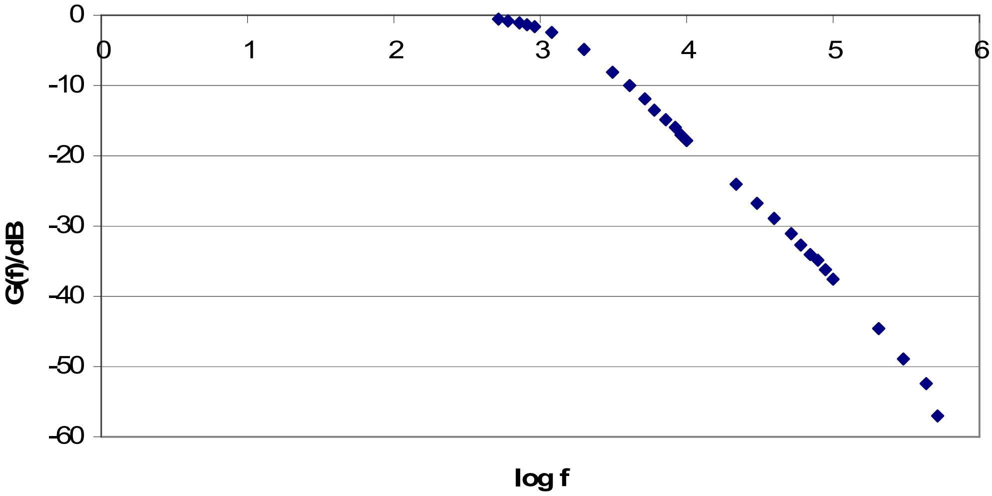
f/Hz	V_{in}/V	V_{out}/V	G(*f*)/dB	log x
510,48	2,1	2	-0,42378598	-0,44185597
602,455	2,1	1,9	-0,86931388	-0,36991008
710,727	2,1	1,85	-1,10095133	-0,29813188
811,827	2,1	1,8	-1,33893579	-0,24037121
902,2	2,1	1,75	-1,58362492	-0,19453187
1189,96	2,1	1,6	-2,36198624	-0,07430233
1994,83	2,1	1,2	-4,86076097	0,150071194
3103	2,1	0,825	-8,11530692	0,341947079
4046,87	2,1	0,66	-10,0535072	0,457284557
5102,83	2,1	0,54	-11,7965107	0,557976403
6067,3	2,1	0,44	-13,5753324	0,633160773
7107,5	2,1	0,38	-14,848714	0,701882172
8241,2	2,1	0,34	-15,8148076	0,766155757
9085,5	2,1	0,3	-16,9019608	0,808514136
10028,5	2,1	0,268	-17,88169	0,851401282
21597,9	2,1	0,13	-24,1655188	1,184576829
30555	2,1	0,095	-26,8899138	1,335247591
39847,6	2,1	0,075	-28,9431606	1,450567472
51058,3	2,1	0,058	-31,175826	1,558231654

60150,36	2,1	0,048	-32,8195611	1,629403534
69929,9	2,1	0,042	-33,9794001	1,69482821
79547,6	2,1	0,038	-34,848714	1,750792385
89567	2,1	0,032	-36,3413863	1,802313331
99637,66	2,1	0,028	-37,5012253	1,848588823
205578,2	2,1	0,0125	-44,5061856	2,163142362
302000	2,1	0,0075	-48,9431606	2,330172246
439287	2,1	0,005	-52,4649858	2,492913654
514000	2,1	0,003	-56,9019608	2,561128422

The theoretical value of fg was calculated with formula (13). The resistance was 9450 Ohm and the capacity was 10 nF.

$f_g = 1684.1793 Hz$

Not standardized Bode plot for determination of f_g.



The graphically determination of the barrier frequency was done by calculating the intersection between the x-axis and the straight line with the equation y=20.058 + 63.18.

The phase shift φ was calculated with following formula:

$$\varphi = 2\pi f \Delta t \,.$$