Determination of the speed of sound in Helium and Nitrous oxide (CPV)

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Abstract: In the following experiment the speed of sound in helium and nitrous oxide was measured. Tree different methods were used; the first was the amplitude maximum method. The speed of sound for Helium is 832 ± 27 m/s, for Nitrous oxide 261 ± 3 m/s. The second method was the phase difference, the velocity for the sound in Helium is 805 ± 15 m/s and in Nitrous oxide 266 ± 5 m/s. The third experiment was the method of plotting Lissajous-figures, the velocity of sound in Helium is determined as 843 ± 5 m/s and in Nitrous oxide 267 ± 1 m/s.
The second part was the determination of the sampling frequency for a white noise using a Fourier-transformation in both gas, which was 4 Hz and the Nyquist-frequency was 5000 Hz for both gases.

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1. Introduction

1.1. Theory

Standing wave

Two waves traveling in opposite directions can build up, a so called standing wave, caused by the interference between them. The outcome of this is the following wave equation:

$$\frac{\partial^2}{\partial t^2}\xi(\vec{r},t) = v^2\left(\frac{\partial^2\xi(\vec{r},t)}{\partial x^2} + \frac{\partial^2\xi(\vec{r},t)}{\partial y^2} + \frac{\partial^2\xi(\vec{r},t)}{\partial z^2}\right) \quad (1)$$

The resonance condition of a standing wave with maximum amplitude in a tube with the length L is:

$$L = \frac{(2\cdot n+1)\cdot\lambda}{4} \quad (2)$$

The wave length can be calculated by measuring the distance between the amplitudes maximums. The peak-to-peak distance is always $\frac{\lambda}{2}$.

Speed of sound for the ideal gas

The spread of sound in a medium is connected to a change of pressure and density. So the wave speed can be expressed by:

$$v^2 = \frac{1}{\kappa_S\cdot\rho}, \quad \kappa = \frac{1}{v^2\cdot\rho} \quad (3)$$

The compressibility under adiabatic or isothermal conditions is defined as:

$$\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S \quad (4a)$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad (4b)$$

The relation between the adiabatic and isothermal compressibility is:

$$\frac{\kappa_S}{\kappa_T} = \frac{C_p}{C_V} = \gamma \quad (5)$$

C_p: isobar thermal capacity
C_v: isochor thermal capacity

With the ideal gas equation and the isothermal compressibility connected to formula (3) follows:

$$v_s^{id} = \sqrt{\gamma\frac{R\cdot T}{M}} \quad (6)$$

Lissajous figures

If two signals with the same frequency are plotted with the help of an oscilloscope, the resulting figure is called Lissajous figures. By varying the phase difference of the signals the form of the figures changes too. The form of the figures changes periodic with 2π relating to the wave length.

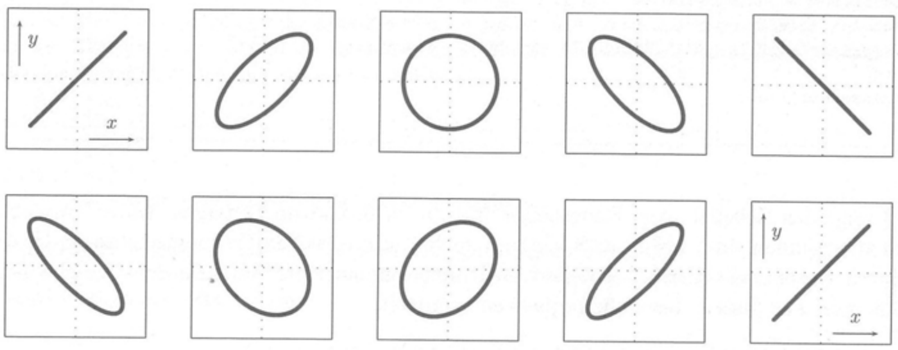

Abb. 1: Lissajous-figures starting from the phase difference 0, π/4, π/2, 3 π/4, π, ... , 2 π.

Noise stimulation and Fourier-Transformation

Following the Fourier-theorem every periodical signal can be added up as infinite sum of Cosinus-vibrations:

$$s(t) = \sum_{n=0}^{\infty} C_n \cos(2\pi n f_0 t + \phi_n) \quad \textbf{(7)}$$

In the experiment the signal is measured in sequence of discrete data points with are followed in a certain distance of time. The time T determines the base frequency and the distance between the discrete frequencies:

$$f_0 = \Delta f = \frac{1}{T} \quad \textbf{(8)}$$

The sampling rate or sampling frequency is determined by:

$$f_{samp} = \frac{1}{\Delta t} \quad \textbf{(9)}$$

The reciprocal value of the time distance between two measurements precise the highest possible part of the frequency f_{max} in the Fourier-row:

$$f_{\max} = \frac{N}{2 \cdot \Delta f} = \frac{N}{2 \cdot T} = \frac{f_{samp}}{2} \quad \textbf{(10)}$$

For the Calculation of the Fourier-row a Matlab-script as appended in the appendix (see section 5.4) was used.

In the Fourier-transformation-plot frequencies which are in resonance at that certain length of the tube have a maximum in amplitude. Out of these frequency, with:

$$f_n = \frac{(2 \cdot n - 1) \cdot v}{4 \cdot L} \quad \textbf{(11)}$$

and:

$$v = \frac{f_n \cdot 4 \cdot L}{2 \cdot n - 1} = \frac{f_{n+1} \cdot 4 \cdot L}{2(n+1) - 1} \quad \textbf{(12)}$$

n can be calculated, which shows the multiple of the ground-resonance, the frequency f_n is.

2. Experiment

2.1. Experimental setup

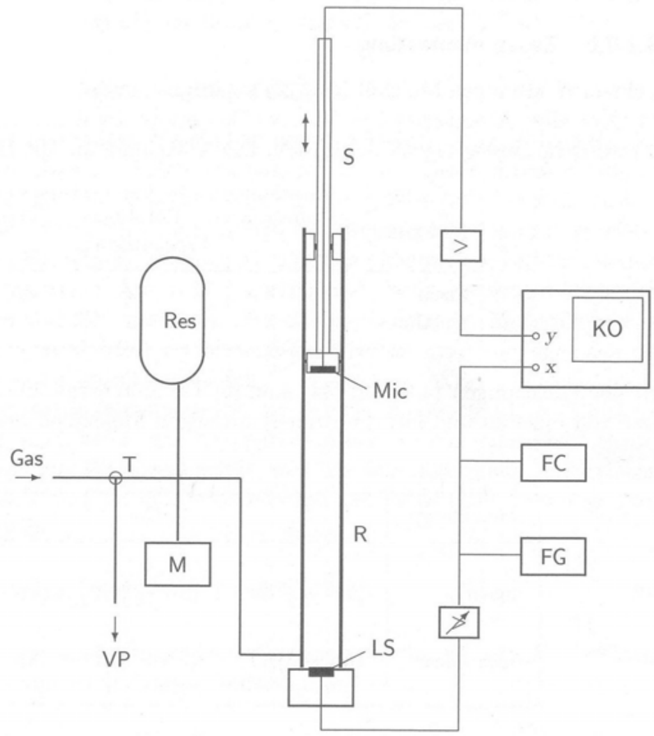

Abb. 2: Schematically picture of the experimental setup for the determination of the speed of sound. FC is the counter for the frequency (Metex Universal-System MS-9140), FG the frequency generator, KO is the oscilloscope (Kikusui COS 2020), LS the speaker, M is the digital manometer (Druck Ltd. DPI-700), Mic is the microphone, R the glass tube, Res the reservoir for the gas, S the stamp, T is the three-way valve, VP the vacuum-pump (Büchi V-503) and > is the amplifier (PC-ETH). [1]

The used setup is presented in Abb. 2. The frequency generator generates a sinus-like wave with a constant frequency or white noise for the experiment with the Fourier-transformation. The signal of the frequency-generator is divided into two; one signal goes to the speaker and the other directly to the oscilloscope. The direct input of the frequency generator and the output of the microphone can be observed at the oscilloscope. The speaker sends the signal through the glass tube, which can be filled with different gases and the length of the tube can be varied by using the stamp.

2.2. Substances

Helium	M: 4 g/mol Density: 0,1785 kg/m^3 S: 9-23
Nitrous oxide	M: 34 g/mol Density: 1,85 kg/m^3 R: 8 S: 9-17

Tab. 1: Chemical information of the Substance which were used during the experiment

2.3. Execution of the experiment

For the change of gas the glass tube was evacuated with the vacuum pump, than washed with the gas from a balloon and evacuated again and filled with the gas until ambient pressure was established.

3.1.2. The standing wave

The experiment was divided into three different parts, in the first part, a certain frequency (He: 7.135 kHz; N_2O: 4.044 kHz) ran through the glass tube filled with either Helium or Nitrous oxide and the stamp was moved in steps of 0.5 cm up and down (He: 34.3 cm – 54.8cm and from 50cm – 34 cm; N_2O: 20 cm – 30 cm and from 29.6 cm – 19.6 cm) and the amplitude, measured in Volt, of the wave was noted down.
In the second part the input of the frequency generator and output of the microphone were compared on the oscilloscope. The stamp was moved until the wave coming from the microphone had the same phase as the signal directly from the frequency generator. The values of the length of the glass tube were noted down.
In the third part the input of the frequency generator was plotted against the output of the microphone, resulting a Lissajous-figure. The length of the tube was noted down, when the phase difference was a multiple of π.
Every part was measured twice, once going up the length and then afterwards going down, with each gas.

3.1.2. Fourier-transformation

In stead of a certain frequency a white noise, containing a whole bunch of different frequency's, were send through a glass tube with a constant length. The noise was displayed on the oscilloscope were the plot could be clipped so the noise could be transferred on to the computer to save it. Then the noise was released and clipped again, in total 5 times for every gas for three different lengths. With a Matlab-script the plots were Fourier-transformed and then evaluated.

3. Analysis and Results

3.1. Method of standing wave

The speed of sound was calculated with the formula (6) for the ideal gas. For the determination of the speed of sound the equation $v = f \cdot \lambda$ was used. The theoretical value of γ was calculated with the equation (5), under assumption of an ideal gas. Equation (6) was used for calculation γ from the experimental results.

3.1.2. Helium

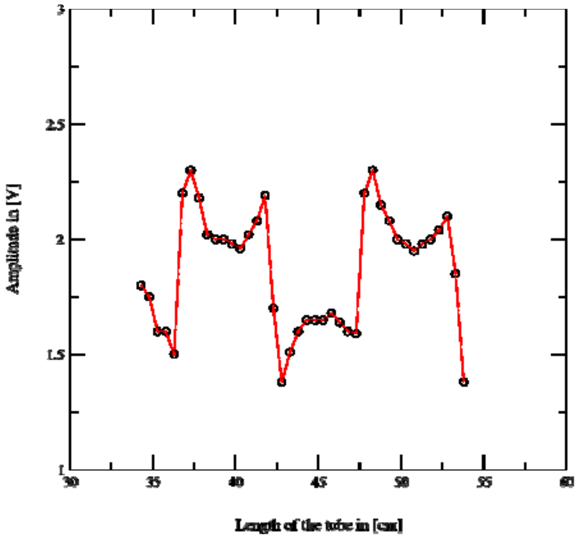

Abb. 3: Plot of the amplitude against the length of the tube (upstream)

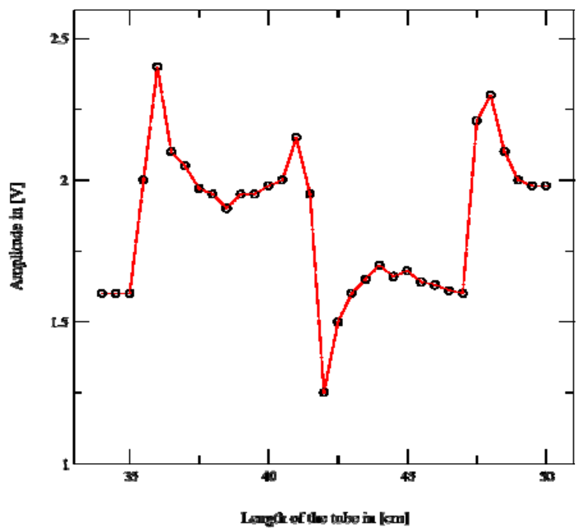

Abb. 4: Plot of the amplitude against the length of the tube (downstream)

Method	*V [m/s]*	γ	C_v [J/(mol K)]
Amplitude maximum	832 ± 27	1.136 ± 0.074	18.3 ± 1.4
Phase difference	805 ± 15	1.66 ± 0.04	12.496 ± 0.089
Lissajous	843 ± 5	1.166 ± 0.014	17.823 ± 0.045
Literature value	965 [1]	1.67 [2]	12.4715 [2]

Tab. 2: Calculated values for Helium, T = 20 °C, p = 963 mbar

The theoretical values of C_v, C_p, γ and the velocity of sound for the ideal gas:

C_p [J/(mol K)]	20.79
C_v [J/(mol K)]	12.47
γ	1⅔
V [m/s]	1007.76

Tab 3: Theoretical values for Helium as ideal gas.

3.1.2. Nitrous oxide

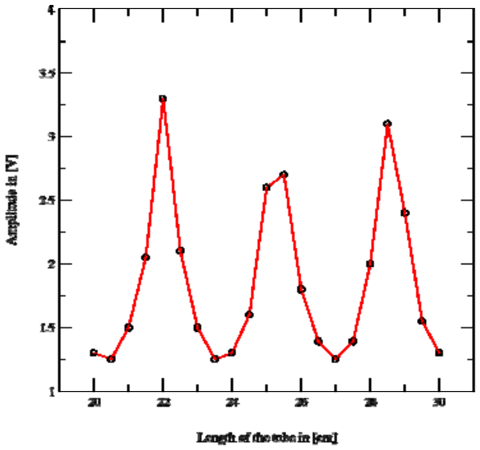

Abb. 5: Plot of the amplitude against the length of the tube (upstream)

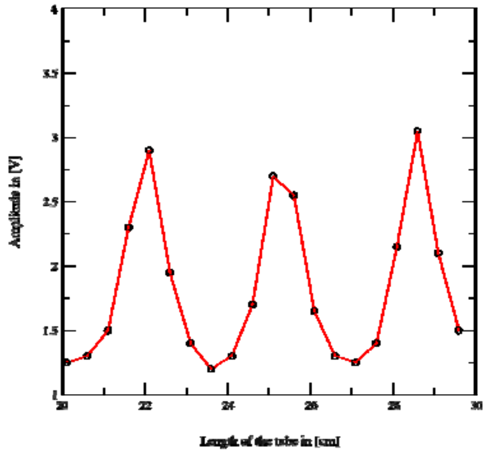

Abb. 6: Plot of the amplitude against the length of the tube (downstream)

Method	*V [m/s]*	γ	C_v [J/(mol K)]
Amplitude maximum	261 ± 3	1.23 ± 0.028	31.21 ± 0.51
Phase difference	266 ± 5	1.277 ± 0.048	30 ± 1.3
Lissajous	267 ± 1	1.2869 ± 0.0096	29.82 ± 0.05
Literature value	263 (0°) [2]	1.28 [3]	30.06 [3]

Tab. 4: Calculated values for Nitrous oxide, T = 22 °C, p = 958 mbar

The theoretical values of C_v, C_p, γ and the velocity of sound for the ideal gas:

C_p [J/(mol K)]	29.10
C_v [J/(mol K)]	20.79
γ	$1\frac{2}{5}$
V [m/s]	278.48

Tab 5: Theoretical values for Helium as ideal gas.

3.2. Fourier-transformation

The figures, generated by Matlab, were analyzed in Matlab and some characteristic peaks were noted down in the plots. n and the velocity of the sound were calculated by using formula (12).

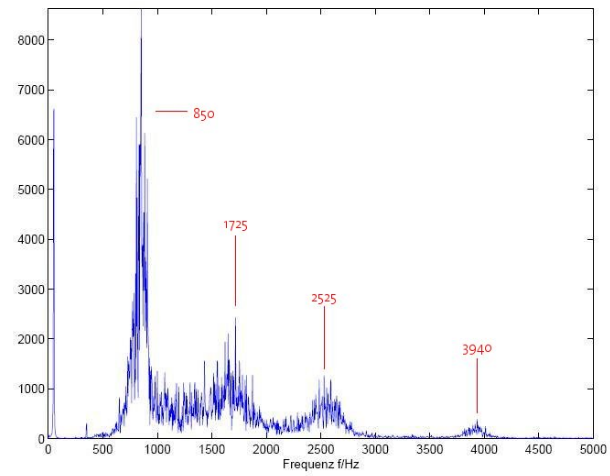

Abb 7: Helium, Length 40cm, T = 20 °C, p = 964 mbar

Peak	Calculated n	Determined n	v_s [m/s]
850		1	
1725	2.656	2	638.8
2525		3	
3940		4	

Tab. 6: Determined values for Helium at Length 40 cm.

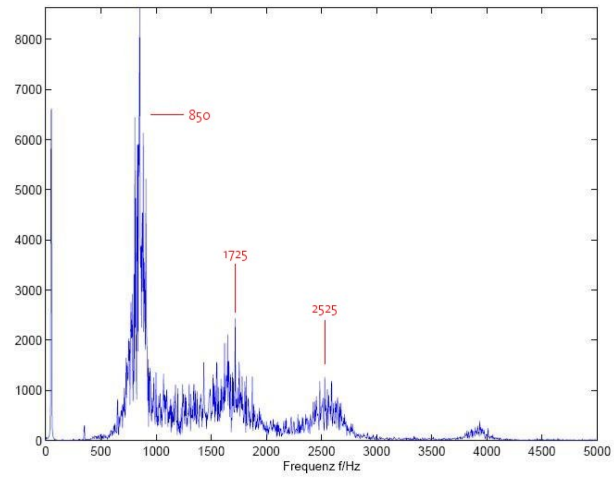

Abb 8: Helium, Length 45cm, T = 20 °C, p = 962 mbar

Peak	Calculated n	Determined n	v_s [m/s]
850		1	
1725	2.656	2	718.8
2525		3	

Tab. 7: Determined values for Helium at Length 45 cm.

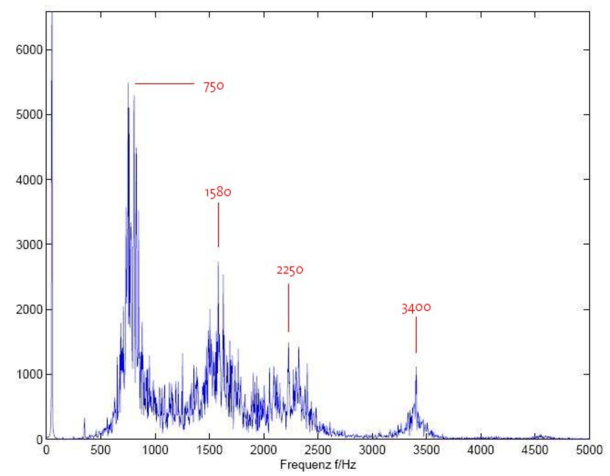

Abb 9: Helium, Length 50cm, T = 20 °C, p = 961 mbar

Peak	Calculated n	Determined n	v_s [m/s]
750		1	
1580	2.456	2	881.9
2250		3	
3400		4	

Tab. 8: Determined values for Helium at Length 50 cm.

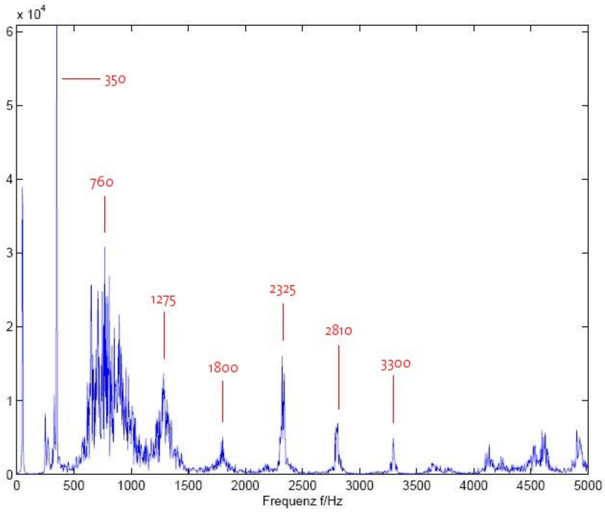

Abb 11: Nitrous oxide, Length 40cm, T = 20 °C, p = 962 mbar

Peak	Calculated n	Determined n	v_s [m/s]
350		1	
760		2	
1275		3	
1800	3.9	4	420
2325		5	
2810		6	
3300		7	

Tab. 9: Determined values for Nitrous oxide at Length 40 cm.

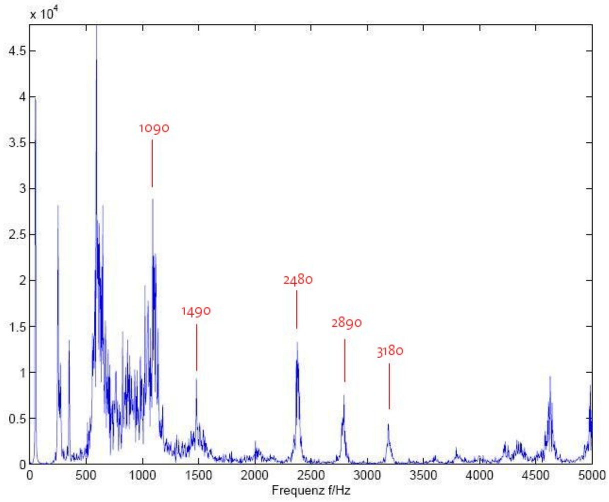

Abb 12: Nitrous oxide, Length 45cm, T = 20 °C, p = 962 mbar

Peak	Calculated n	Determined n	v_s [m/s]
1090		3	
1490		4	
2480	6.5	6	368.9
2890		7	
3180		8	

Tab. 10: Determined values for Nitrous oxide at Length 45 cm.

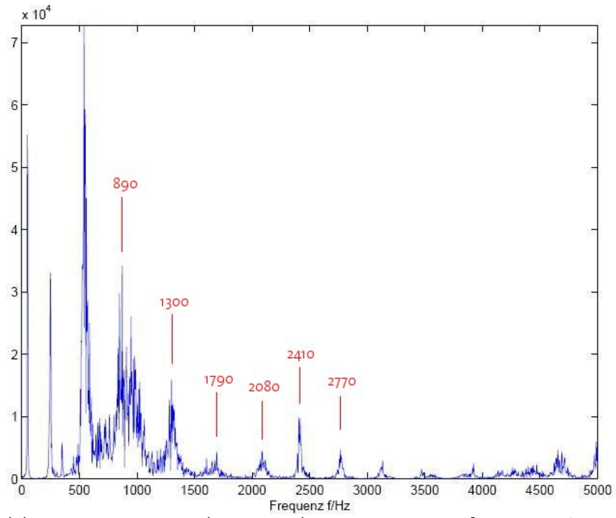

Abb 13: Nitrous oxide, Length 50cm, T = 20 °C, p = 962 mbar

Peak	Calculated n	Determined n	v_s [m/s]
890		3	
1300		4	
1790		5	
2080	6.8	6	330.2
2410		7	
2770		8	

Tab. 11: Determined values for Nitrous oxide at Length 50 cm.

The Nyquist-frequency can be calculated out of (10). Because N (the amount of time steps, in our case 2500) and T (the period of the measurement, in our case 250 ms) wasn't change in between the experiments, it's the same for both gases:

$$f_{\max} = f_{Nyquist} = \frac{N}{2 \cdot T} = \frac{2500}{2 \cdot 250ms} = 5000Hz$$

The sampling rate or sampling frequency is determined by (8). Its stays the same for both gases, for the same reasons as mentioned above.

$$f_0 = \Delta f = \frac{1}{T} = \frac{1}{250ms} = 4Hz$$

4. Discussion

The determination of the speed of sound with different methods has shown big differences in precision and practical use. The Method of the stationary wave was not very precisely. This is made clear by the difference in the results of the error calculation. The determination of the maximum amplitude was only a estimation. So it was not possible to determine the precise value. The error of the comparison of the phase differences is smaller than the error of the method of standing wave. It shows that the method of standing wave is more practical. The method of phase differences showing Lissajous-figures is the precisest method. The periodic change of the form of the figures allows an accurate determination of the phase difference. This is why this method shows the smallest errors by the determination of the speed of sound.
The Fourier-transformation experiment didn't show up any difficulty, the only Problem was that the Matlab script, which was delivered, wasn't accurate because the program wavestar provided the number of the measured value instead of the time stamp. The evaluation of the plots showed a peak at about 50 Hz, which is the frequency of the power net in Europe.
The measurement with the gas Helium provided some problems. Two labor spaces were using the same vacuum pump, which caused that the helium was mixed up with the other gas the other lab space used (S. Bachmann and S. Breitler, Gas: Argon). Therefore velocity of the sound was in general too low. The calculated values for were as well not very accurate in comparison to the values found in the literature. The theoretical values and the calculated values of C_v were the same. The reason for this accordance is that Helium is a one-atom-gas that behaves very similar to the ideal gas.
The measurements with nitrous oxide were more accurate than the measurements with the gas Helium, because this time the vacuum pump wasn't used at the same time and the valve were closed. The values for and C_v were close to the literature values. The theoretical values of C_v don't go with the founded values, because nitrous oxide consists out of three atoms, which meant that nitrous oxide doesn't behave exactly like a ideal gas (stretch vibrations which were added up are just approximations).
The calculation of the Nyquist frequency and the sampling frequency did not depend on the kind of gas which was used, because the just depended on the time frame in which the data was saved.

5. Appendix

5.1. Experimental raw data

5.1.1. Helium

L [cm]	U [V]
34,3	1,8
34,8	1,75
35,3	1,6
35,8	1,6
36,3	1,5
36,8	2,2
37,3	2,3
37,8	2,18
38,3	2,02
38,8	2
39,3	2
39,8	1,98
40,3	1,96
40,8	2,02
41,3	2,08
41,8	2,19
42,3	1,7
42,8	1,38
43,3	1,51
43,8	1,6
44,3	1,65
44,8	1,65
45,3	1,65
45,8	1,68
46,3	1,64
46,8	1,6
47,3	1,59
47,8	2,2
48,3	2,3
48,8	2,15
49,3	2,08
49,8	2
50,3	1,98
50,8	1,95
51,3	1,98
51,8	2
52,3	2,04
52,8	2,1
53,3	1,85
53,8	1,38

L [cm]	U [V]
50	1,975
49,5	1,975
49	2
48,5	2,1
48	2,3
47,5	2,21
47	1,6
46,5	1,61
46	1,63
45,5	1,64
45	1,68
44,5	1,66
44	1,7
43,5	1,65
43	1,6
42,5	1,5
42	1,25
41,5	1,95
41	2,15
40,5	2
40	1,98
39,5	1,95
39	1,95
38,5	1,9
38	1,95
37,5	1,97
37	2,05
36,5	2,1
36	2,4
35,5	2
35	1,6
34,5	1,6
34	1,6

Distance [cm]	Average [cm]	STDEV [cm]
5,46	5,69	0,243036348
5,5		
5,9		
5,9		

Tab. A1: Noted down values for Helium up- and downstream for the maximum amplitude.

L [cm]	Difference [cm]	Average [cm]	STDEV [cm]
24,2		11,5333333	0,40792156
36,41	12,21		
48,05	11,64		
59,2	11,15		
70,5	11,3		
81,65	11,15		
70,5	11,4		
59,1	11,1		
48	11,9		
36,1	11,95		
24,15			

Tab. A2: Noted down values for Helium up- and downstream for the phase difference.

L [cm]	Difference [cm]	Average [cm]	STDEV [cm]
30,1		5,75882353	0,13945798
36,1	6		
41,85	5,75		
47,65	5,8		
53,4	5,75		
59,15	5,75		
64,9	5,75		
70,5	5,6		
76,2	5,7		
81,6	5,4		
76,15	5,75		
70,4	5,7		
64,7	5,75		
58,95	5,8		
53,15	5,65		
47,5	5,9		
41,6	5,9		
35,7	5,95		
29,75			

Tab. A3: Noted down values for Helium up- and downstream for the Lissajous-figures.

5.1.2. Nitrous oxide

L [cm]	U [V]
20	1,3
20,5	1,25
21	1,5
21,5	2,05
22	3,3
22,5	2,1
23	1,5
23,5	1,25
24	1,3
24,5	1,6
25	2,6
25,5	2,7
26	1,8
26,5	1,39
27	1,25
27,5	1,39
28	2
28,5	3,1
29	2,4
29,5	1,55
30	1,3

L [cm]	U [V]
20	1,3
20,5	1,25
21	1,5
21,5	2,05
22	3,3
22,5	2,1
23	1,5
23,5	1,25
24	1,3
24,5	1,6
25	2,6
25,5	2,7
26	1,8
26,5	1,39
27	1,25
27,5	1,39
28	2
28,5	3,1
29	2,4
29,5	1,55
30	1,3

Distance [cm]	Average [cm]	STDEV [cm]
3,3	3,225	0,05
3,2		
3,2		
3,2		

Tab. A4: Noted down values for N_2O up- and downstream for the maximum amplitude.

L [cm]	Difference [cm]
24,7	
31,15	6,45
37,81	6,66
44,41	6,6
51	6,59
57,55	6,55
64,25	6,7
70,3	6,05
77,5	7,2
70,35	6,1
64,25	6,6
57,65	6,65
51	6,6
44,4	6,6
37,8	6,6
31,2	6,55
24,65	

Average [cm]	STDEV [cm]
6,56785714	0,25905506

Tab. A2: Noted down values for N_2O, up- and downstream for the phase difference.

L [cm]	Difference [cm]
14,75	
18	3,25
21,35	3,35
24,6	3,25
27,95	3,35
31,2	3,25
34,5	3,3
37,8	3,3
41,1	3,3
44,4	3,3
47,7	3,3
51	3,3
47,7	3,25
44,45	3,35
41,1	3,3
37,8	3,3
34,5	3,25
31,25	3,3
27,95	3,34
24,61	3,3
21,31	3,16
18,15	3,45
14,7	

Average [cm]	STDEV [cm]
3,29761905	0,05638304

Tab. A6: Noted down values for N2O up- and downstream for the Lissajous-figures.

5.2. Used equations for error calculation

The following equations were used for calculation of γ and C_v.

$$C_v = \frac{C_p}{\gamma} \quad (13) \qquad v_s^{id} = \sqrt{\gamma \cdot \frac{RT}{M}} \quad (14) \qquad v_s^2 \cdot \frac{M}{RT} = \gamma \quad (15)$$

The resulting error propagation was calculated by common equation:

$$s_z^2 = \left(\frac{\partial z}{\partial a}\right)^2 \cdot (s_a)^2 \quad (16)$$

(17) is the equation for error propagation for the experimental determined velocity of sound. The error of the frequency was 10 Hz.
(18) is the error propagation for γ and equation (19) gives the error propagation for C_v.

$$s_v^2 = \lambda^2 \cdot s_f^2 + f^2 \cdot s_\lambda^2 \quad (17)$$

$$s_\gamma^2 = (2v_s \cdot \frac{M}{RT})^2 \cdot (s_v)^2 \quad (18)$$

$$s_{C_v}^2 = \left(\frac{C_p}{\gamma^2}\right)^2 \cdot s_\gamma^2 \quad (19)$$

5.3. Raw Data for Fourier-transformation

If wished, they can be downloaded at http://www.crew.li/cpv

5.4. MatLab Script

```
clear;
n = 2500;
timeperdivision = 0.025;
spec = zeros(1,n);
gn = input('Dateigrundname: ','s');
anz = input('Anzahl Dateien: ');
ext = '.txt';
outfilename = strcat(gn,'spec.dat');

for i = 1:anz
   i_string = num2str(i);
   infilename = strcat(gn,i_string,ext);
   disp(infilename);
   infile = fopen(infilename, 'r');
   [data,N] = fscanf(infile, '%e', [2,Inf]);
   fclose(infile);
   t = data(1,:);
   t = (t/2500)*10*timeperdivision;
   s = data(2,:);
   s = s - mean(s);
   y = fft(s);
   m = abs(y);
   p = m .* m;
   spec = spec + p;
end;
T = max(t);
f_NQ = n/(2*T);
f = linspace(0,f_NQ,n/2);

spec = spec(1:n/2);
plot(f,spec);
axis([0, f_NQ, 0, max(spec)]);
xlabel('Frequenz f/Hz');
outfile = fopen(outfilename, 'w');
for i = 1:(n/2)
   fprintf(outfile, '%10.7e %10.7e \n', f(i), spec(i));
end;
fclose(outfile);
```

5.5. Literature

[1] E. Meister, *Grundpraktiukum Physikalische Chemie*; vdf Hochschulverlag AG; 2006
[2] *Handbook of Chemistry and Physics*; CRC Press; 2003
[3] www.nist.org, 01.04.2007